Harvard Undergraduate Science Olympiad India
2026 Final Round
7th-8th Grade
Mathematics Section: Exam

INSTRUCTIONS:

The HUSO India Final Round Math section consists of two sections. All questions are multiple choice and have one unique best answer. Section I consists of largely independent multiple choice questions, while Section II contains questions tied to particular thematic ideas.

You may spend 90 minutes on the Mathematics section. **You are NOT allowed a calculator. No additional notes or electronics are allowed.**

All answers must be bubbled on the provided on the answer sheet. Any writing on the exam booklet outside the designated boxes in the answer sheets will not be graded. You may write in this booklet, but **NO WRITING IN THIS BOOKLET WILL BE GRADED.**

It is to your benefit to recall that *questions are not ordered by difficulty*!

Grading:

- Section I: Single Select questions; +1 point per correct answer, -.25 points per incorrect answer, 0 points if question left blank.
- Section II: Multiple Select questions; +2 point per correct answer, -0.5 points per incorrect answer, 0 points if question left blank.
- If there are ties, the higher Section I score wins.

Do your best! Embrace and conquer the challenge!

#	Section	Questions	% of Total Points
1	General Mathematics questions	30	42.9
2	Themed questions	20	57.1
	Total	**50**	**100**

Section 1: General Mathematics

1) What is the product of the roots of the polynomial $p(x) = 2x^4 - 8x^3 - 43x^2 + 4x + 21$?

a. 2
b. -2
c. 21/2
d. -21/2
e. 21

2) How many of the roots of the polynomial $p(x) = 2x^4 - 8x^3 - 43x^2 + 4x + 21$ are rational?

a. 0
b. 1
c. 2
d. 3
e. 4

3) Alice has a favourite sequence whose first term is 1. For all terms but the first in this sequence, a term is either double the previous term if the previous term was odd or exactly one greater than the previous term if the previous term was even. What is the sum of the digits of the 20th term in this sequence?

a. 2
b. 5
c. 8
d. 12
e. 16

4) Bob has two arithmetic sequences, A and B. Sequence A has first two terms $4,\ 9$ and sequence B has first two terms $3,\ 5$. Carl has two geometric sequences, C and D. Sequence C has first two terms $4,\ 6$ and sequence D has first two terms $3,\ 2$. What is the smallest index n for which the sum of the n^{th} terms of Carl's sequences are greater than the sum of the n^{th} terms of Bob's sequences?

a. 6
b. 7
c. 8
d. 9
e. 10

5) Choose two positive real numbers $a,\ b$. Let AM be their arithmetic mean, GM be their geometric mean, and HM be their harmonic mean. Over all choices of $a,\ b$, what is the maximum value of $AM/GM\ -\ GM/HM$?

a. 0
b. 1
c. $\sqrt{2}$
d. Finite value, none of the above.
e. ∞

6) A polynomial P(x) with integer coefficients satisfies P(x+1) – P(x) = x^2 for all integers x, and P(0) = 0. What is P(10)?

a. 285
b. 330
c. 385
d. 440
e. 495

7) A polynomial P(x) satisfies P(x) + P(–x) = x^4 + 1. What is the coefficient of x^3 in P(x)?

a. -1
b. 0
c. 1
d. No such P exist
e. There are infinitely many possible coefficients.

8) Let n be a positive integer. Define $f(n) = \gcd(n^2 + 1, n + 1)$. What are all possible values of $f(n)$?

a. 1 only
b. 1 and 2
c. 1 and 3
d. 1, 2, and 3
e. All positive integers

9) How many integers n with $1 \le n \le 2024$ satisfy that $n^2 + n + 1$ is divisible by 7?

a. 0
b. 288
c. 289
d. 577
e. 578

10) Let p be a prime strictly greater than 3. How many of the following 4 expressions are always divisible by 24?

- $p^2 - 1$
- $p^3 - 1$
- $p^4 - 1$
- $p^5 - 1$

a. 0
b. 1
c. 2
d. 3
e. 4

11) How many ordered pairs of (not necessarily positive) integers (x, y) satisfy $x^2 + y^2 = 2025$?

a. 8
b. 12
c. 16
d. 24
e. 32

12) Let n be a positive integer. Suppose n has exactly three positive divisors. Which of the following must be true?

a. n is prime
b. n is a square of a prime
c. n is twice a prime
d. n is a product of two primes
e. n is a perfect cube

13) Let n be the smallest positive integer such that n! ends in exactly 100 zeros. What is the remainder when the sum of the digits of n is divided by 5?

a. 0
b. 1
c. 2
d. 3
e. 4

14) For how many integers n with $1 \le n \le 500$ does gcd(n, 500) = gcd(n + 2, 500)?

a. 150
b. 200
c. 250
d. 300
e. 400

15) The number 2025 is written in base b as 121_b. What is b?

a. 42
b. 43
c. 44
d. 45
e. 46

16) Let a and b be integers such that a^2 + b^2 is divisible by 13. Which of the following must be true?

a. a is divisible by 13
b. b is divisible by 13
c. a – b is divisible by 13
d. a + b is divisible by 13
e. None of the above are necessarily true.

17) In triangle ABC, let the midpoints of sides AB and AC be D and E, respectively. Suppose line segments CD and BE were the same length. Which of the following statements MUST be true about triangle ABC?

- Statement 1: Angle A is 60 degrees
- Statement 2: Triangle ABC is isosceles
- Statement 3: Triangle ABC is obtuse

a. Statement 1 only
b. Statement 2 only
c. Statement 3 only
d. Statements 2 and 3 only
e. All statements must be true.

18) A point P lies in equilateral triangle ABC, at a distance 3 from side AB and a distance 5 from side AC. The minimal area of triangle ABC can be expressed as $x\sqrt{3}/3$, where x is a positive integer. What is the sum of the digits of x?

a. 4
b. 6
c. 8
d. 10
e. 12

19) Circle O is externally tangent to both coordinate axes and the line 3x+4y = 12, and is entirely contained within the region bounded by these three lines. Circle Ω is externally tangent to the x axis, the line 3x+4y=12, and circle O.
What is the radius of circle O?

a. $1/2$
b. $\sqrt{2}/2$
c. 1
d. $\sqrt{2}$
e. 2

20) The radius of circle Ω can be written as $\frac{\sqrt{x}-1}{\sqrt{x}+1}$ for some x. What is the remainder when x is divided by 5?

a. 0
b. 1
c. 2
d. 3
e. 4

21) David has an equiangular hexagon (a convex hexagon with all interior angles equal to 120° but varying side lengths) ABCDEF. He knows AB = 3, BC = 8, CD = 2, and DE = 5. What is the remainder when EF + AF is divided by 5?

a. 0
b. 1
c. 2
d. 3
e. 4

22) David actually has a lot of equiangular hexagons, all with different combinations of side lengths. To help reduce the number of side lengths per hexagon to remember, he decides to memorize the lengths of n sides of each hexagon, chosen uniformly randomly. What is the smallest value n for which if David knows n sides of an equiangular hexagon, he can always determine all 6 of its sides?

a. 2
b. 3
c. 4
d. 5
e. David must memorize all the sides of each hexagon.

23) In triangle ABC, a line through the centroid parallel to BC intersects AB and AC at D and E. What fraction of the area of triangle ABC lies inside triangle ADE?

a. 2/9
b. 1/3
c. 4/9
d. 2/3
e. 8/9

24) Emily draws diagonals in a regular hexagon to form a triangulation, where drawn diagonals do not intersect in the interior. How many triangulations contain at least one diagonal connecting opposite (maximal distance between them) vertices of the hexagon?

a. 8
b. 10
c. 12
d. 14
e. 16

25) In a group of 10 people, each person shakes hands with exactly 5 others. Which statement must be true?

a. Two people shook hands with exactly the same set of people.
b. There exist three people who mutually shook hands.
c. There exist two people who did not shake hands with anyone in common.
d. The graph contains an even number of triangles.
e. None of the above.

26) Fred chooses a subset S of {1, 2, 3, … , 20}. How many subsets have even size and even sum?

a. 2^17
b. 2^18
c. 3 × 2^17
d. 2^19
e. 2^20

27) Seven equally spaced points lie on a circle. How many triangles formed by these points contain the center of the circle?

a. 12
b. 14
c. 16
d. 18
e. 21

28) Each vertex of a square is colored either black or white. How many colorings have the property that no reflection symmetry of the square leaves the coloring unchanged?

a. 2
b. 4
c. 6
d. 8
e. None of the above.

29) How many integer triples (x, y, z) satisfy x + y + z = 0, where each of x, y, z is chosen from {−3, −2, −1, 0, 1, 2, 3}?

a. 37
b. 49
c. 61
d. 73
e. 85

30) Gianna has a convex hexagon whose sides are colored alternately red and blue. She labels the vertices with the numbers 1 through 6, each used exactly once. If every red side must connect two numbers whose sum is even, how many valid labelings exist?

a. 0
b. 24
c. 48
d. 72
e. 144

Section 2: Themed Questions

The greatest common divisor (also known as greatest common factor, GCF, or GCD) of two positive integers *a* and *b* can be defined as the largest positive integer *c* such that *c* divides both *a* and *b*.

We can expand this definition of the GCD to monomials (polynomials with leading coefficient 1) with integer coefficients. For two such monomials p(x), q(x), we can define the GCD of these two monomials as the monomial f(x) of highest degree that divides both p(x) and q(x).

For example, the GCD of p(x) = x^2 - 3x + 2 and q(x) = x^3 - 3x^2 + 3x - 1 is f(x) = x-1.

31) Let f(x) be the GCD of the monomials p(x) = x^3 + 2x^2 - 35x and q(x) = x^3-1. What is the degree of f(x)?

a. 0.
b. 1.
c. 2.
d. 3.
e. The degree of f(x) is strictly greater than 3.

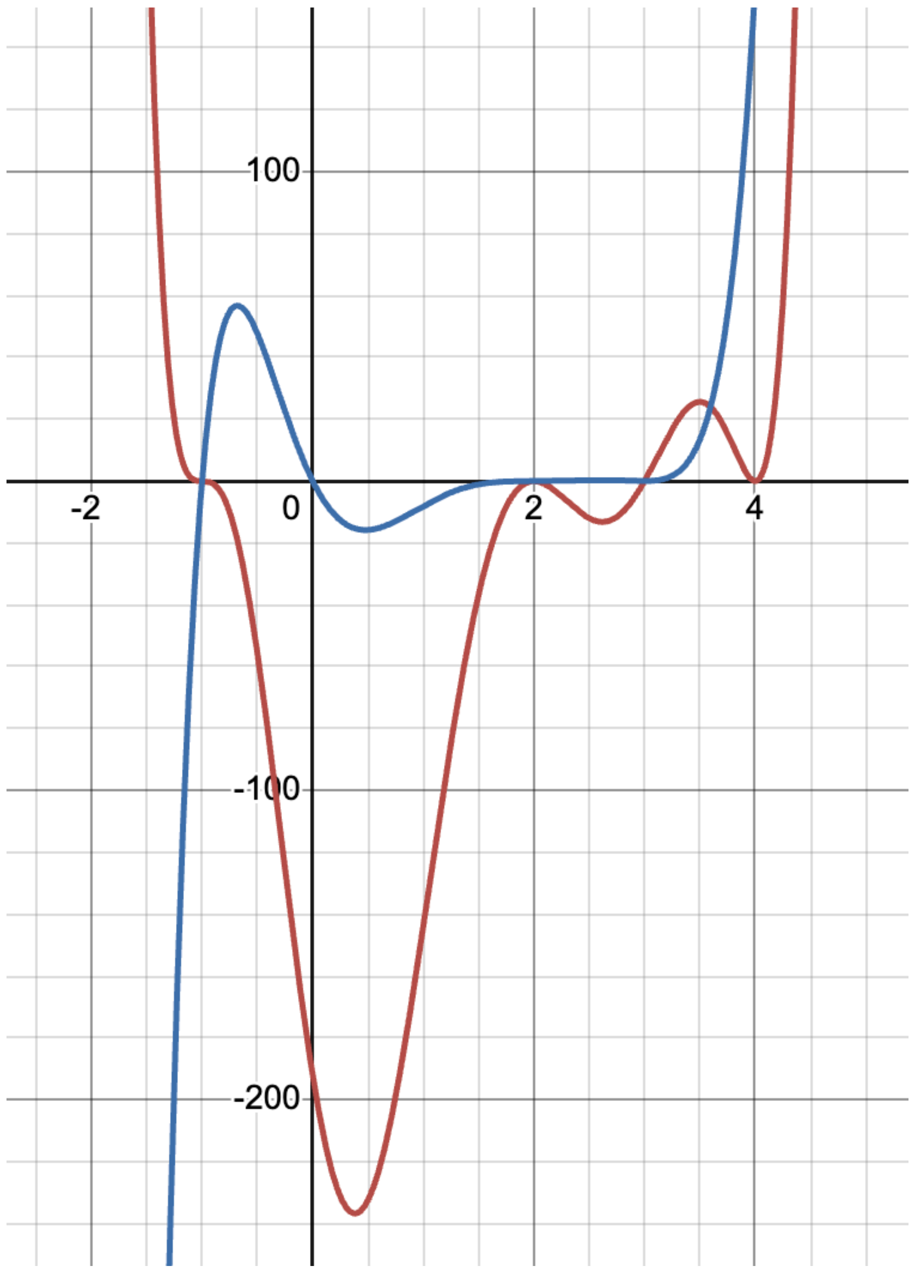
32) There exist many similarities between both definitions of the GCD. Which of the following statements are TRUE about the GCD f(x) of monomials p(x) and q(x)?

- Statement 1: The GCD is unique: there is only one possible f(x) with maximal degree
- Statement 2: The degree of the GCD (the degree of f(x)) is never larger than the GCD of the degrees of p(x) and q(x)
- Statement 3: The GCD of the monomials m(x) = p(x)/f(x) and n(x) = q(x)/f(x) is always the constant polynomial g(x) = 1.

a. Statement 1 only.
b. Statement 2 only.
c. Statements 2 and 3 only.
d. Statements 1 and 3 only.
e. All statements are true.

It can also be useful to think about geometric properties of the GCD of monomials.

Consider the following graphs of the two monomials r(x) and b(x), where r(x) is the red monomial and b(x) is the blue polynomial.

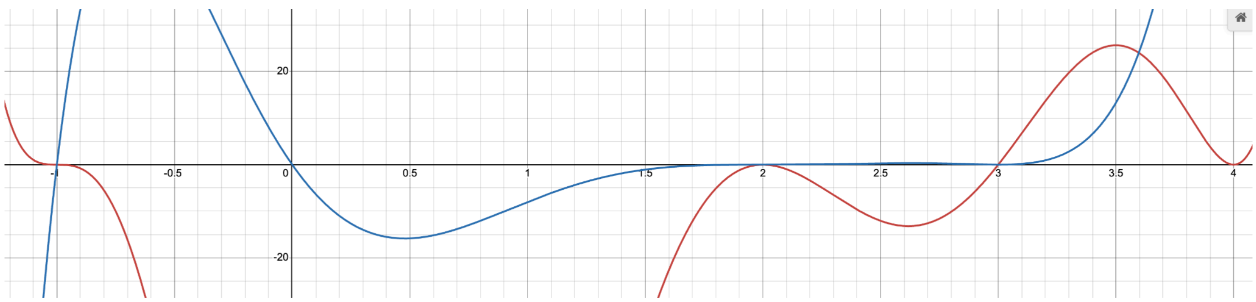


These monomials are both somewhat complicated, but all of their roots are integer and shown in the plot above.

33) You know that each of the polynomials either has degree 7 or degree 8. Which of the following assigns the correct degrees to each of r(x), b(x)?

a. r(x) and b(x) both have degree 7
b. r(x) has degree 7 and b(x) has degree 8
c. r(x) has degree 8 and b(x) has degree 7
d. r(x) and b(x) both have degree 8

To help with the following two questions, here is a close-up of the monomials in the region around y=0:



Note that b(x) is positive strictly between x=2 and x=3, and b(x) = 0 at x=2 and x=3. This may be a little hard to see from the above images.

Let h(x) be the GCD of r(x) and b(x).

34) What is the sum of the DISTINCT divisors of h(x)?

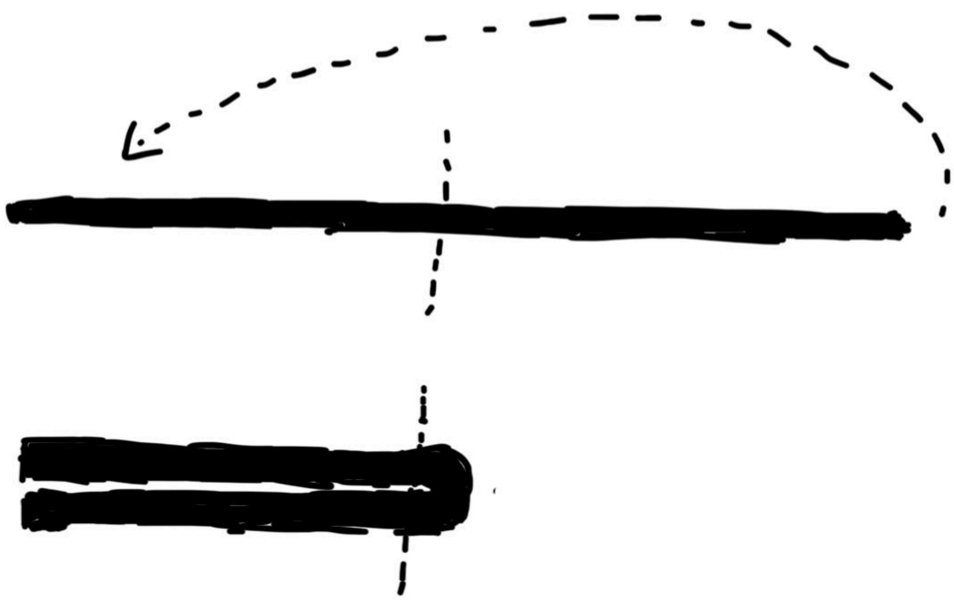
a. -4
b. -2
c. 0
d. 2
e. 4

35) What is h(1)? The multiplicity of each root in r(x) and b(x) is at most 3.

a. -4
b. -2
c. 0
d. 2
e. 4

Origami is a traditional Japanese art which has recently found numerous applications in various engineering disciplines. In origami, one starts with a flat, 2D sheet of (usually square) paper and makes various folds to form a 3D shape. For our exploration, we'll consider all folds to be along straight lines (creases) and that all paper we use has negligible width.

Regular paper is actually quite hard to fold multiple times! Origami paper is much thinner than regular paper.



Let's consider some arbitrary paper with thickness d (and other dimensions far greater than d). Then, when we fold this paper in half, the paper will double in thickness for most of its length, but a small mass of paper will be required to form 'the curve', actually connecting the two layers. While for just one fold, with thin paper, the total amount of paper that is a part of 'the curve' is relatively small, this can be a significant problem if one needs to perform multiple folds, and is part of the reason why it is near impossible to fold a standard piece of paper in half more than 8 times.

36) A well known adage is that if one managed to fold a sheet of printer paper in half 42 times, its final thickness would stretch from the earth to the moon! The distance from the earth to the moon is 400,000 km. Which of the following is closest to the thickness of an unfolded sheet of printer paper?

a. 1 micrometer
b. 10 micrometers
c. 0.1 millimeters
d. 1 millimeter
e. 1 centimeter

37) Suppose 'the curve' of the crease is made as tight and clean as possible, minimizing the mass of paper used to form it. In this case, assuming fixed dimensions for the height and width of the paper, the amount of paper used in the crease is proportional to d raised to what power?

a. 0
b. 1
c. 2
d. 3
e. 4

For the rest of the question, ignore the curve, and suppose that origami paper has 0 thickness. Now, let's use origami to do some geometry! Alice has a square piece of origami paper, with four corners labeled A, B, C, D. She folds corner A onto some point P on CD, creating crease L, which intersects AD at E and BC at F. The side length of the square is 1.

38) Suppose P was the midpoint of CD. What is the length AE?

a. 2/5
b. 3/8
c. 1/2
d. 5/8
e. 3/5

39) Alice chooses some point P such that points F and B coincide. The ratio CP/CD must lie in which of the following intervals (lower bound inclusive, upper bound exclusive)?

a. [0, 0.2)
b. [0.2, 0.4)
c. [0.4, 0.6)
d. [0.6, 0.8)
e. [0.8, 1)

40) Now, Alice marks a point X inside square ABCD. The distance from X to the center of ABCD is 1/4 . Alice then makes two folds with creases both passing through the (original) center of ABCD, making sure that the point X moves after each fold. The two creases intersect at an angle of 45 degrees. After both folds have been made, what is the distance between the final location of X and the final location of the original center of ABCD?

a. 1/4
b. $\frac{\sqrt{2}}{4}$
c. 1/2
d. $\frac{\sqrt{2}}{2}$
e. This cannot be uniquely determined.

The fraction $\frac{1}{7}$ has many interesting properties, one of which is that for all positive integers a strictly less than 7, the fraction $\frac{a}{7}$ has a repeating decimal expansion that is a cyclic shift of the digits in the repeating decimal expansion of $\frac{1}{7}$. For example, the decimal part of $\frac{2}{7} = 0.2857142857142857...$ is the decimal part of $\frac{1}{7} = 0.142857142857142857 ...$ with each digit shifted two digits to the left (ignoring anything to the left of the decimal point).

Let's try to search for some other examples of numbers that have this property! Specifically, we are looking for positive integers n where the fractions $\frac{a}{n}$, where a is a positive integer strictly less than n, have repeating decimal expansions which are cyclic shifts of one another (up to taking fractional parts).

41) Bob made the following conjectures about such numbers n. How many such conjectures are true? Consider only the case when n is a positive integer at least 2.

- No non-prime n have this property
- All prime n have this property
- If n satisfies this property, then the smallest repeating block of digits in $\frac{1}{n}$ is n-1 digits long.
- If n satisfies this property, then all the digits in the smallest repeating block of digits in $\frac{1}{n}$ are distinct

a. 0
b. 1
c. 2
d. 3
e. 4

42) For how many of the following values of n does the property hold?

- 11
- 13
- 15
- 17

a. 0
b. 1
c. 2
d. 3
e. 4

We can also think about a very conceptually related problem stated quite differently!

Call the order mod b of a the smallest value k for which $a^k \equiv 1 \ mod\ b$. If there does not exist any k for which this equivalence is possible, we use the convention that the order is equal to ∞.

43) Over all possible values a, how many distinct (possibly infinite) values of the order are possible if b = 10?

a. 1
b. 2
c. 3
d. 4
e. 5

44) What is the order mod 7 of 10?

a. 6
b. 7
c. 8
d. 9
e. 10

45) Which of the following is equal to the number of repeating digits in the decimal expansion of $\frac{2}{13}$?

a. Order mod 13 of 2
b. Order mod 13 of 10
c. Order mod 10 of 2
d. Order mod 10 of 13
e. Order mod 13 of 20

Binomial coefficients are extremely important to combinatorics, and many complex counting problems can be broken down into sums of binomial coefficients. There exist many identities which can simplify sums and products of binomial coefficients, many of which stem from problems in combinatorics which admit multiple solutions.

46) What is the sum $\binom{10}{0} + \binom{10}{2} + \binom{10}{4} + \binom{10}{6} + \binom{10}{8} + \binom{10}{10}$?

a. 256
b. 512
c. 1024
d. 2048
e. None of the above.

Combinatorially, the sum in question 4a can be conceptualized as the total number of ways to pick an even number of (distinguishable) objects out of a set of 10 total objects. However, there may exist alternative viewpoints which are easier to explicitly calculate.

47) The sum $\binom{3}{3} + \binom{4}{3} + \binom{5}{3} + \binom{6}{3} + \binom{7}{3} + \binom{8}{3} + \binom{9}{3} + \binom{10}{3}$ can be written as $\binom{a}{b}$ for some a and b, with $b \leq \frac{a}{2}$. What is the remainder when a+b is divided by 5?

a. 0
b. 1
c. 2
d. 3
e. 4

48) Which of the following is a correct combinatorial interpretation of the sum in the above question?

a. The total number of ways to choose 3 objects from a set of 10 distinguishable objects.
b. The total number of ways to pick the largest element in a subset and the 3 extra objects in the subset from an ordered set of 10 distinguishable objects.
c. The total number of ways to choose 4 objects from a set of 10 indistinguishable objects.
d. The total number of ways to pick the largest element in a subset and the 3 extra objects in the subset from an ordered set of 10 indistinguishable objects.
e. None of the above are valid interpretations.

49) The sum $\binom{9}{3}\binom{8}{0} + \binom{9}{2}\binom{8}{1} + \binom{9}{1}\binom{8}{2} + \binom{9}{0}\binom{8}{3}$ is equal to $\binom{a}{b}$ for some a and b, with $b \leq \frac{a}{2}$. What is the remainder when a+b is divided by 5?

a. 0
b. 1
c. 2
d. 3
e. 4

50) Which of the following is a correct combinatorial interpretation of the sum in the above question?

a. Given 17 objects split into two groups of 9 and 8, the total number of ways to choose 3 objects between both groups.
b. Given 17 objects split into two groups of 9 and 8, the total number of ways to choose 3 objects from each group.
c. Given a group of 9 people, the total number of ways to choose 8 team members and 3 team captains.
d. Given a group of 9 people, the total number of ways to choose 8 team members and at most 3 team captains.
e. None of the above are valid interpretations.