

# Section I: Mathematics

**Suggested Time:** 30 minutes

**Number of Problems:** 12 Questions

**Instructions:** For each of the following questions, select the correct answer and fill in the corresponding bubble on the answer sheet. Bubbling in multiple answers per question will result in an automatic incorrect answer. You get +1 point for a correct answer, 0 points for a blank answer, and -0.25 points for an incorrect answer. You may move freely between this section and other sections.

- (Algebra) A student is told that the sum of 2024 distinct whole numbers is 2049198. What is the product of these same 2024 distinct natural numbers?
  - The product is 0;
  - The product is the square of the sum of the numbers;
  - The product is  $2024!$ ;
  - The product is the sum of the square of the numbers;
  - We cannot uniquely determine the product.
- (Algebra) A rectangular prism has length  $\ell$ , width  $w$ , and height  $h$ . We know that the volume of the prism is 12, and the surface area is 18. What is the value of  $\frac{1}{\ell} + \frac{1}{w} + \frac{1}{h}$ ?
  - $\frac{1}{2}$ ;
  - $\frac{3}{16}$ ;
  - $\frac{9}{16}$ ;
  - $\frac{3}{4}$ ;
  - $\frac{9}{4}$ ;
- (Algebra) Determine the value of  $\frac{2}{0!+1!+2!} + \frac{3}{1!+2!+3!} + \dots + \frac{2024}{2022!+2023!+2024!}$ .
  - $1 - \frac{1}{2023!}$ ;
  - $1 - \frac{1}{2024!}$ ;
  - $\frac{1}{2} - \frac{1}{2023!}$ ;
  - $\frac{1}{2} - \frac{1}{2024!}$ ;
  - 2024;
- (Number Theory) The numbers  $2^{2024}$  and  $5^{2024}$  are written one after the other in the base 10. How many digits does the resulting number have?
  - The number of digits is 2022;
  - The number of digits is 2023;
  - The number of digits is 2024;
  - The number of digits is 2025;

- (e) We cannot uniquely determine the number of digits.
5. (Number Theory) Let a *backward-prime* be a number such that, when the digits are read in backward order, the resulting number is a prime. Find the sum of the smallest and largest two-digit numbers  $N$  such that  $N$  and  $2N$  are both backward-primes.
- (a) 150;  
 (b) 134;  
 (c) 115;  
 (d) 132;  
 (e) 114;
6. (Number Theory) A student writes the numbers  $1, 2, 3, \dots, 2023, 2024$  concatenated on a board to get the number  $S = 12345678910111213 \dots 202220232024$ . Which of the following divisibility conditions are satisfied by  $S$ ?
- (a)  $2 \nmid S, 3 \nmid S$ , and  $9 \nmid S$ ;  
 (b)  $2 \mid S, 3 \nmid S$ , and  $9 \nmid S$ ;  
 (c)  $2 \mid S, 3 \mid S$ , and  $9 \nmid S$ ;  
 (d)  $2 \mid S, 3 \mid S$ , and  $9 \mid S$ ;  
 (e)  $2 \nmid S, 3 \mid S$ , and  $9 \mid S$ ;
7. (Geometry) In triangle  $ABC$ ,  $D$  is the foot of the altitude from  $A$  to  $BC$ . The circles inscribed in the triangles  $ABD$  and  $ACD$  are tangent to the line  $AD$  at the points  $E$  and  $F$  respectively. If the sides  $AB, AC$ , and  $BC$  have lengths 1, 2, and 3, find the length of the segment  $EF$ .
- (a)  $\frac{1}{2}$ ;  
 (b)  $\frac{1}{4}$ ;  
 (c)  $\frac{1}{8}$ ;  
 (d)  $\frac{1}{16}$ ;  
 (e) We cannot uniquely determine the length of  $EF$ .
8. (Geometry) A regular hexagon  $A_1A_2A_3A_4A_5A_6$  has side length 6. We shrink this hexagon by a factor of  $2/3$  to get the hexagon  $B_1B_2B_3B_4B_5B_6$  such that the centres of the hexagons are the same and  $B_iB_{i+1}$  is parallel to  $A_iA_{i+1}$  for  $i = 1, 2, \dots, 5$ . What is the area of trapezoid  $A_1B_1B_2A_2$ ?
- (a)  $4\sqrt{3}$ ;  
 (b)  $5\sqrt{3}$ ;  
 (c)  $10\sqrt{3}$ ;  
 (d)  $12\sqrt{3}$ ;  
 (e)  $15\sqrt{3}$ ;

9. (Geometry) In triangle  $ABC$ , the sides  $AB$ ,  $AC$ , and  $BC$  all have lengths 2024. If  $P$  is a random point in the interior, what is the sum of the distances of  $P$  towards the sides of the triangle?
- (a) The sum of the distances is 1012;
  - (b) The sum of the distances is  $1012\sqrt{3}$ ;
  - (c) The sum of the distances is 2024;
  - (d) The sum of the distances is  $2024\sqrt{3}$ ;
  - (e) We cannot uniquely determine the sum of the distances.
10. (Combinatorics) A student needs to receive 37 cents worth of change. If we assume there are an infinite number of 1-cent, 5-cent, and 20-cent coins, in how many possible ways can the student get the 37 cents worth of change?
- (a) 8;
  - (b) 12;
  - (c) 15;
  - (d) 18;
  - (e) 22;
11. (Combinatorics) In a box there are 2024 balls numbered from 1 to 2024. First, we remove the balls divisible by 4. Then, we remove those divisible by 11. How many balls are there left in the box?
- (a) 1428;
  - (b) 1426;
  - (c) 1380;
  - (d) 1334;
  - (e) 1332;
12. (Combinatorics) A rabbit stands at point 0 on the real line axis. At the  $i$  move, the rabbit chooses to jump either to the left or to the right a distance of  $2i + 1$ . Can the rabbit get back to point 0 on the real line axis after 2023 moves?
- (a) Yes, only if on the first move it jumps to the right;
  - (b) Yes, only if on the first move it jumps to the left;
  - (c) Yes, only if it jumps back and forth at each step;
  - (d) Yes, it can go back to point 0, no matter in which direction the first move;
  - (e) No, will never go back to point 0, no matter the direction it jumps at each move;