Section I: Mathematics

Suggested Time: 30 minutes

Number of Problems: 12 Questions

Instructions: For each of the following questions, select the correct answer and fill in the corresponding bubble on the answer sheet. Bubbling in multiple answers per question will result in an automatic incorrect answer. You get +1 point for a correct answer, 0 points for a blank answer, and -0.25 points for an incorrect answer. You may move freely between this section and other sections.

1. (Algebra) Let $x_1, x_2, \ldots, x_{2024} \in \mathbb{R}^+$ such that $\frac{x_1}{\sqrt{1\cdot 2}} + \frac{x_2}{\sqrt{2\cdot 3}} + \ldots + \frac{x_{2024}}{\sqrt{2024\cdot 2025}} = 2024$. Find the minimal value of $x_1^2 + x_2^2 + \ldots + x_{2024}^2$.

   (a) 2023^2;
   (b) $2023 \cdot 2024$;
   (c) 2024^2;
   (d) $2024 \cdot 2025$;
   (e) 2025^2.

2. (Algebra) Determine the value of $\sqrt{1 + \frac{1}{1^2} + \frac{1}{2^2}} + \sqrt{1 + \frac{1}{2^2} + \frac{1}{3^2}} + \ldots + \sqrt{1 + \frac{1}{2023^2} + \frac{1}{2024^2}}$.

   (a) $2023\frac{2022}{2023}$;
   (b) $2023\frac{2023}{2024}$;
   (c) $2024\frac{2023}{2024}$;
   (d) $2024\frac{2024}{2025}$;
   (e) 2024;

3. (Algebra) Let there some numbers written on the board such that the square of any written number is greater than the product of any other 2 written numbers. Find the maximum number of numbers written on the board.

   (a) 1;
   (b) 2;
   (c) 3;
   (d) 4;
   (e) There could be infinite numbers.

4. (Number Theory) How many three digit numbers that are divisible by 7 contain the digit 3?

   (a) 30;
   (b) 32;
   (c) 34;
   (d) 37;

(e) 39;

5. (Number Theory) Determine the number of zeroes in which the number $n = 1001 \cdot 1002 \cdots 2022 \cdot 2023 \cdot 2024$ ends.

   (a) 113;
   (b) 224;
   (c) 249;
   (d) 254;
   (e) 503;

6. (Number Theory) Let $(a_n)_{n \geq 1}$ be a sequence of natural numbers satisfying the property $(a_n, a_m) = (n, m)$ for all $m \neq n$, where (a, b) represents the greatest common divisor of a and b. What is the value of a_{2025}?

   (a) 2023;
   (b) 2024^2;
   (c) 2024;
   (d) 2025^2;
   (e) 2025;

7. (Geometry) In triangle ABC, $m(\angle C) = 60°$. If $AC = 8$, find the sum of all possible $AB + BC$ with AB and BC being positive integers.

   (a) 28;
   (b) 40;
   (c) 56;
   (d) 66;
   (e) 84;

8. (Geometry) Three circles with radii 1, 2, and 3 are externally tangent to each other in pairs at points A, B, and C. Find the area of the triangle ABC.

   (a) $\frac{3}{5}$;
   (b) $\frac{6}{5}$;
   (c) $\frac{5}{3}$;
   (d) $\frac{5}{6}$;
   (e) We cannot uniquely determine the area of ABC.

9. (Geometry) Let rectangle $ABCD$ have side lengths $AB = 10$ and $BC = 2 + 2\sqrt{3}$. Circle ω, centered at point O, meets $ABCD$ at four points E, F, G, H such that E and F are on the same side and G and H are on the opposite side. Suppose that $\angle EOF = 120°$ and the radius of ω is 4. Then, the area that is inside $ABCD$ but outside ω can be written in the form of $a + b\sqrt{c} + d\pi$ for integers a, b, c, d. Find $a + b + c + d$.

(a) 19;
(b) 27;
(c) 36;
(d) 43;
(e) 48;

10. (Combinatorics) How many trapezoids, with trapezoids including rectangles, can be formed with four vertices of a regular octagon?

    (a) 32;
    (b) 30;
    (c) 28;
    (d) 26;
    (e) 24;

11. (Combinatorics) Kevin wants to park a red car, silver car, and blue car in a parking lot with 8 parking spaces. However, he's a really bad parker, so he wants to allocate at least one space in between each of the cars in case he needs it. How many ways can he park the three cars?

    (a) 336;
    (b) 270;
    (c) 120;
    (d) 90;
    (e) 48;

12. (Combinatorics) Amy wants to travel from the bottom left corner to the top right corner of a 5×5 grid by using only right and up moves. What's the expected number of squares below such paths she wants to take?

    (a) 12.5;
    (b) 10;
    (c) 15
    (d) 7.5
    (e) 17.5